

# AMRNodeElliptic User Guide: on rectangular problem domains

Peter McCorquodale

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# Chapter 1

## Introduction

This document describes an extension to the Chombo package [CGL<sup>+</sup>00] for solving elliptic equations using adaptive mesh refinement on multiple levels with *node*-centered data. See the `AMRNodeElliptic` design document [McC02] for a description of the algorithms used.

Chapter 2 describes the class `NodeFArrayBox` and functions that use this class to manipulate node-centered data. Interfaces for elliptic equation solvers are described in chapter 3. Some internal classes of the solvers are described in chapter 4. User interfaces for physical boundary conditions are described in chapter 5.

The `AMRNodeElliptic` package requires that Chombo library be installed. See chapter 1 of the Chombo design document [CGL<sup>+</sup>00] for requirements and installation of Chombo.

### 1.1 Constructing a makefile with `AMRNodeElliptic`

This section specifies how to construct a `GNUmakefile` compatible with the `AMRNodeElliptic` extension to Chombo. Most of this is also described in section 1.2.3 in the Chombo design document [CGL<sup>+</sup>00].

To make an external makefile, first say where Chombo is installed by specifying `CHOMBO_HOME` and the base name of the application by specifying `ebase`.

```
#####define the location of Chombo
CHOMBO_HOME = ../../../../ChomboLib
```

```
#####define the base name of the application
ebase := myProg
```

Then specify the names of the Chombo libraries that should be linked:

```
##
## names of Chombo libraries needed by this program, in order of search.
##
LibNames := AMRTools BoxTools
```

Now specify the path and source files external to Chombo and AMRNodeElliptic:

```
## path to look for source files not in Chombo
VPATH_LOCAL = .
INCLUDES_LOCAL = -I.

## C++ sources go here
CXX_SOURCES = NodePoissonBC.cpp testFuncs.cpp testRunPoint.cpp

## Chombo Fortran sources go here
FORT_SOURCES = testProb.ChF

## standard C sources go here
C_SOURCES = myc1.c myc2.c

## standard Fortran sources go here
F_SOURCES = myfort1.f myfort2.f
```

So far, the description of the makefile is as for any makefile that uses the Chombo library. When using AMRNodeElliptic, end the makefile with the following lines:

```
#####define location of AMRNodeElliptic source directory
SRC_DIR = ../../src
## these commands include the AMRNodeElliptic source
include $(SRC_DIR)/Make.package
INCLUDES_LOCAL += -I$(SRC_DIR)
VPATH_LOCAL += $(SRC_DIR)

## end of makefile for use with Chombo
include $(CHOMBO_HOME)/mk/Make.example
```

# Chapter 2

## Node-Centered Data

### 2.1 The class NodeFArrayBox

The NodeFArrayBox class is a wrapper for a node-centered FArrayBox. This class is used in LevelData<NodeFArrayBox> with an underlying cell-centered DisjointBoxLayout.

The reason for introducing a new NodeFArrayBox class, instead of just using node-centered FArrayBoxes, is to allow the use of LevelData. A LevelData<FArrayBox> based on node-centered FArrayBoxes would have an underlying DisjointBoxLayout of node-centered boxes, and no two of the boxes could be adjacent, because if they were, they would share points, and the layout would not be disjoint. To get around this problem, we use a cell-centered DisjointBoxLayout and build a LevelData<NodeFArrayBox> on it, with the data located at the surrounding nodes of the cell-centered grids. Data at nodes shared by two or more grids is stored redundantly in all of the corresponding NodeFArrayBoxes.

The API for NodeFArrayBox is as follows.

- NodeFArrayBox()

Default constructor. User must subsequently call define.

- NodeFArrayBox(  
    const Box& a\_box,  
    int        a\_nComp = 1)

Creates a NodeFArrayBox object.

**Inputs:**

- a\_box specifies the cell-centered box. The data lie on the surrounding nodes of this box.
- a\_nComp specifies the number of components of the FArrayBox.

- define(

```

    const Box& a_bx,
    int         a_nComp = 1)

```

Defines a NodeFArrayBox object. Arguments are the same as those of the constructor above.

- FArrayBox& getFab()

Returns a reference to the node-centered FArrayBox containing the data.

- const Box& box()

Returns the cell-centered box of which the surrounding nodes contain the data.

- Real norm(

```

    const Real a_dx,
    const int  a_p = 2,
    const int  a_startComp = 0,
    const int  a_numComp = 1) const

```

For  $a_p > 0$ , returns the  $L^{a-p}$  norm of the data in `a_numComp` components, starting at component number `a_startComp`. The norm is computed by integration using the trapezoidal rule with mesh spacing `a_dx`. For  $a_p = 0$ , returns the  $L^\infty$  norm of the same data. An overloaded version of this function performs its operations over a subbox.

- Real maxnorm(

```

    const int a_startComp = 0,
    const int a_numComp = 1) const

```

Returns the maximum value of the data in `a_numComp` components, starting at component number `a_startComp`. An overloaded version of this function performs its operations over a subbox.

### Principal data members

- Box `m_box`. The cell-centered box of which the surrounding nodes contain the data.
- FArrayBox `m_fab`. The node-centered data.

## 2.2 Notes on `LevelData<NodeFArrayBox>::copyTo`

The API for `LevelData<NodeFArrayBox>` is described in the Chombo design document [CGL<sup>+</sup>00], but one part of it deserves particular attention because it does not work in the same way.



```

void copyTo(
    const Interval& a_srcComps,
    BoxLayoutData<T>& a_dest,
    const Interval& a_destComps) const

```

```

void copyTo(
    const Interval& a_srcComps,
    LevelData<T>& a_dest,
    const Interval& a_destComps) const

```

The `LevelData<NodeFArrayBox>::copyTo` function must be used with caution. If the source `LevelData` and the destination, `a_dest`, do not have the same underlying grid layout, then it is possible that some data will not be copied from the source to the destination. The reason is that, in determining points where data should be copied, `copyTo` finds intersections of the underlying *cell*-centered boxes and then copies at the surrounding nodes. See Figure 2.1 for an example in which data are not copied at some of the nodes shared by source and destination.

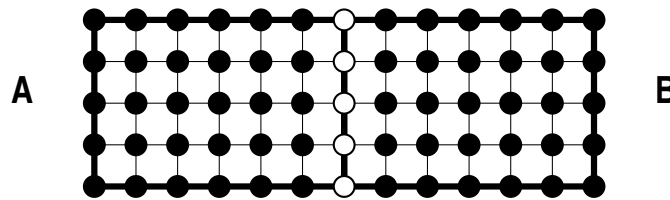


Figure 2.1: Suppose A and B are `LevelData<NodeFArrayBox>`s, each having an underlying grid layout of a single grid, and these two grids abut, as shown above where the nodes of intersection are indicated by hollow circles. Then the statement `A.copyTo(A.interval(), B, B.interval())` results in *no* data being copied from A to B, even though they share nodes, because the two grids have no *cells* in common.

If all cells in the grid layout of the destination of the `copyTo` are also in the grid layout of the source, then this problem of uncopied data does not arise. For example, when copying all of the data on one AMR level to a coarsened version of the grids on a finer AMR level, `copyTo` works as expected, because of the nesting requirement for AMR levels.

But whenever we want to use `LevelData<NodeFArrayBox>::copyTo` from `src` where the underlying grid layout of the destination `dest` may contain cells that are not in the grid layout of this, we use a `Copier` object, as follows:

```

Copier myCopier(src.getBoxes(), dest.getBoxes(), IntVect::Unit);
src.copyTo(srcComps, dest, destComps, myCopier);

```

## 2.3 Node-centered norms

### 2.3.1 Norms of valid data on a single level

- `Real norm(const LevelData<NodeFArrayBox>& a_phi,`  
    `const ProblemDomain& a_domain,`  
    `const DisjointBoxLayout* a_finerGridsPtr,`  
    `const int a_nRefFine,`  
    `const Real a_dx,`  
    `const Interval a_comps,`  
    `const int a_p,`  
    `bool a_verbose = true)`
- `Real norm(const LevelData<NodeFArrayBox>& a_phi,`  
    `const Box& a_domain,`  
    `const DisjointBoxLayout* a_finerGridsPtr,`  
    `const int a_nRefFine,`  
    `const Real a_dx,`  
    `const Interval a_comps,`  
    `const int a_p,`  
    `bool a_verbose = true)`

Returns the norm of valid data on one level.

**Inputs:**

- `a_phi` is the data at this level.
- `a_domain` is the entire cell-centered problem domain at this level.
- `a_finerGridsPtr` is a pointer to the grid layout at the next finer level, or NULL if there is no finer level.
- `a_nRefFine` is the refinement ratio between this level and the next finer level.
- `a_dx` is the mesh spacing at this level.
- `a_comps` specifies the components over which to take the norm.
- `a_p` specifies which norm to take: if `a_p = 0` then the  $\max(L^\infty)$  norm, and if `a_p > 0` then the  $L^{a-p}$  norm.
- `a_verbose` specifies whether to display the norm of each grid.

Overloaded versions of this function include arguments with the exterior boundary nodes of this level, and the interior boundary nodes of the coarsened finer-level grids.

- `Real maxnorm(const LevelData<NodeFArrayBox>& a_phi,`  
    `const ProblemDomain& a_domain,`

```

const DisjointBoxLayout*    a_finerGridsPtr,
const int                   a_nRefFine,
const Interval              a_comps,
bool                        a_verbose = true)

```

- Real maxnorm(const LevelData<NodeFArrayBox>& a\_phi,
const Box& a\_domain,
const DisjointBoxLayout\* a\_finerGridsPtr,
const int a\_nRefFine,
const Interval a\_comps,
bool a\_verbose = true)

Returns the max ( $L^\infty$ ) norm of valid data on one level. Inputs are a subset of those for the norm function above, and there are overloaded versions as with norm.

### 2.3.2 Norms of data on multiple levels

- Real norm(
const Vector<LevelData<NodeFArrayBox>\* >& a\_phi,
const Vector<ProblemDomain>& a\_domain,
const Vector<int>& a\_nRefFine,
const Real a\_dxCrse,
const Interval a\_comps,
const int a\_p,
const int a\_lBase,
bool a\_verbose = true)
- Real norm(
const Vector<LevelData<NodeFArrayBox>\* >& a\_phi,
const Vector<Box>& a\_domain,
const Vector<int>& a\_nRefFine,
const Real a\_dxCrse,
const Interval a\_comps,
const int a\_p,
const int a\_lBase,
bool a\_verbose = true)

Returns the norm of valid data on multiple levels.

**Inputs:**

- a\_phi is a vector of pointers to data at multiple levels.
- a\_domain specifies the entire cell-centered problem domain at all levels of resolution. Vector index corresponds to level number.

- `a_nRefFine` specifies the refinement ratios between adjacent levels. `a_nRefFine[ilev]` is the refinement ratio between levels `ilev` and `ilev+1`. `a_nRefFine[ilev]` must be a power of two, with `a_nRefFine[ilev] ≥ 1`.
  - `a_dxCrse` is the mesh spacing at level `a_lBase`.
  - `a_comps` specifies the components over which to take the norm.
  - `a_lBase` is the index of the coarsest level on which the norm is to be computed.
  - `a_p` specifies which norm to take: if `a_p = 0` then the max ( $L^\infty$ ) norm, and if `a_p > 0` then the  $L^{a-p}$  norm.
  - `a_verbose` specifies whether to display the norm of each grid.
- `Real maxnorm(`  
`const Vector<LevelData<NodeFArrayBox>* >& a_phi,`  
`const Vector<ProblemDomain>& a_domain,`  
`const Vector<int>& a_nRefFine,`  
`const Interval a_comps,`  
`const int a_lBase,`  
`bool a_verbose = true)`
  - `Real maxnorm(`  
`const Vector<LevelData<NodeFArrayBox>* >& a_phi,`  
`const Vector<Box>& a_domain,`  
`const Vector<int>& a_nRefFine,`  
`const Interval a_comps,`  
`const int a_lBase,`  
`bool a_verbose = true)`

Returns the max ( $L^\infty$ ) norm of valid data on multiple levels. Inputs are a subset of those for the multi-level norm function above.

## 2.4 Dot product for node-centered data

The `DotProductNodes` functions return the dot product of data distributed over two instances of `LevelData<NodeFArrayBox>` with the same layout. The dot product is the sum, over all valid nodes and over specified components, of the product of the data in the two data holders. That is, for data sets  $f$  and  $g$  on nodes  $\Omega_N$ , the dot product is

$$f \cdot g = \sum_{i \in \Omega_N, \text{valid}} f_i g_i.$$

- `Real DotProductNodes(`  
`const LevelData<NodeFArrayBox>& a_dataOne,`  
`const LevelData<NodeFArrayBox>& a_dataTwo,`

```

const ProblemDomain&      a_domain,
const Interval&          a_comps)

```

- Real DotProductNodes(

```

const LevelData<NodeFArrayBox>& a_dataOne,
const LevelData<NodeFArrayBox>& a_dataTwo,
const Box&                      a_domain,
const Interval&                 a_comps)

```

Returns the dot product of data on one level.

**Inputs:**

- a\_dataOne contains the first set of data on a level.
- a\_dataTwo contains the second set of data on a level.
- a\_domain is the cell-centered problem domain on this level.
- a\_comps specifies the components to be used in computing the dot product.

An overloaded version of this function includes the exterior boundary nodes of the level as an argument. It is more efficient to use this other version when the dot product is to be computed multiple times on data with the same layout.

## 2.5 Functions for interior and exterior boundary nodes

An *interior node* of a grid layout is any node in the layout with the property that all of the cells adjacent to the node are also contained in the grid layout. An *interior boundary node* of a grid layout is an interior node that lies on the boundary of one or more grids in the layout. See Figure 2.2 for a sample layout with interior boundary nodes marked.

An *exterior node* or *exterior boundary node* of a grid layout is any node of the layout that is not an interior node. See Figure 2.3 for the same layout as Figure 2.2, with exterior nodes marked.

### 2.5.1 interiorBoundaryNodes functions

The `interiorBoundaryNodes` functions return the interior boundary nodes of a `DisjointBoxLayout`. The interface would be simplest if the function returned a reference to an `IntVectSet`, but because of the way that `IntVectSet` is implemented, this would be very slow. Instead, the first `interiorBoundaryNodes` function returns a reference to a `LayoutData< Vector<IntVectSet> >` such that the union of all the `IntVectSets` in the `Vectors` contains exactly the indices of all interior boundary nodes of the input layout.

To make matters even more complicated, there is a second `interiorBoundaryNodes` function, which takes two `DisjointBoxLayout` arguments. This function returns a reference to a `LayoutData` that stores the indices of interior boundary nodes of one layout

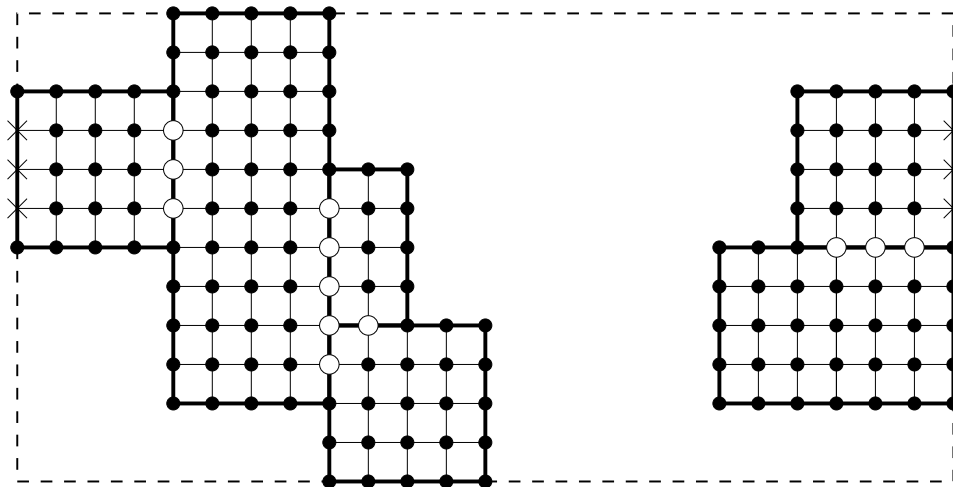


Figure 2.2: Hollow circles indicate the interior boundary nodes of a two-dimensional grid layout inside a problem domain delineated by the dashed lines. Nodes marked  $\times$  are also interior boundary nodes if the problem domain is periodic in the horizontal direction.

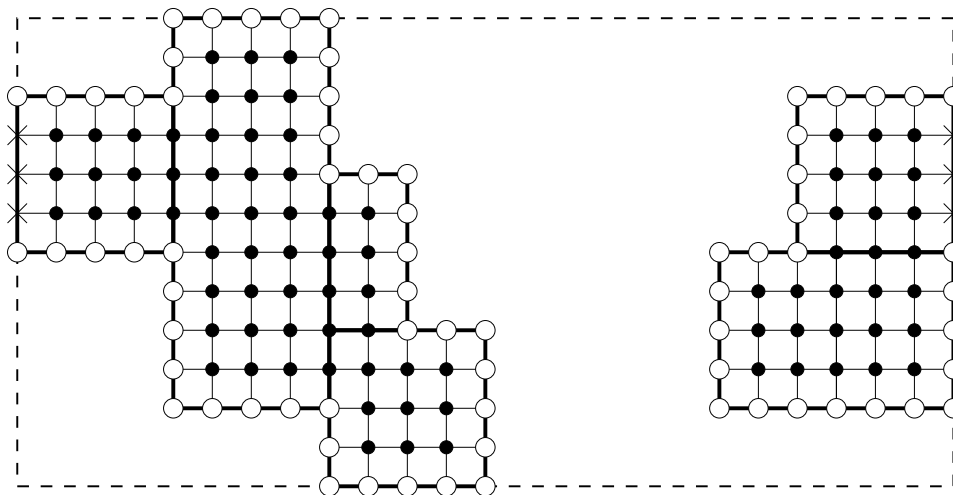


Figure 2.3: Hollow circles indicate the exterior boundary nodes of a two-dimensional grid layout inside a problem domain delineated by the dashed lines. Nodes marked  $\times$  are also exterior boundary nodes if the problem domain is *not* periodic in the horizontal direction. Any node that is not an exterior boundary node is an interior node.

that are also nodes (interior or exterior) of a second layout. The output of this function is needed for calling `copyInteriorNodes`, described in section 2.5.3.

- `void interiorBoundaryNodes(`  
    `LayoutData< Vector<IntVectSet> >& a_IVSV,`  
    `const DisjointBoxLayout& a_boxes,`  
    `const ProblemDomain& a_domain)`
- `void interiorBoundaryNodes(`  
    `LayoutData< Vector<IntVectSet> >& a_IVSV,`  
    `const DisjointBoxLayout& a_boxes,`  
    `const Box& a_domain)`

Returns the interior boundary nodes of a layout.

**Inputs:**

- `a_boxes` specifies the cell-centered layout of boxes.
- `a_domain` specifies the cell-centered problem domain that contains the layout.

**Output:**

- `a_IVSV` contains the interior boundary nodes of `a_boxes` in a `LayoutData` based on `a_boxes`. For each box in the layout of `a_boxes`, there is a `Vector` of `IntVectSets` that together contain the indices of the interior boundary nodes of the box.

- `void interiorBoundaryNodes(`  
    `LayoutData< Vector<IntVectSet> >& a_IVSV,`  
    `const DisjointBoxLayout& a_dest,`  
    `const DisjointBoxLayout& a_src,`  
    `const ProblemDomain& a_domain)`
- `void interiorBoundaryNodes(`  
    `LayoutData< Vector<IntVectSet> >& a_IVSV,`  
    `const DisjointBoxLayout& a_dest,`  
    `const DisjointBoxLayout& a_src,`  
    `const Box& a_domain)`

Returns the interior boundary nodes of a source layout that are also nodes of a destination layout. This function is used when copying from data based on the source layout to data based on the destination layout.

**Inputs:**

- `a_dest` specifies the cell-centered layout of boxes on which to base the `LayoutData` to be returned.

- `a_src` specifies the cell-centered layout of boxes of which we wish to find the interior boundary nodes.
- `a_domain` specifies the cell-centered problem domain that contains both layouts.

**Output:**

- `a_IVSV` contains the interior boundary nodes of `a_src` in a `LayoutData` based on the layout in `a_dest`. For each box in the layout of `a_dest`, there is a `Vector` of `IntVectSets` that together contain the indices of all nodes of the box that are also interior boundary nodes of `a_src`.

If `a_dest` is the same as `a_src`, then the result is the same as calling the first `interiorBoundaryNodes` function with `a_boxes` set to `a_dest`.

### 2.5.2 exteriorBoundaryNodes function

- `void exteriorBoundaryNodes(
 LayoutData< Vector<IntVectSet> >& a_exterior,
 const LayoutData< Vector<IntVectSet> >& a_interior,
 const DisjointBoxLayout& a_boxes)`

Returns the exterior boundary nodes (also called exterior nodes) of `a_boxes`.

**Inputs:**

- `a_interior` is the object returned by `interiorBoundaryNodes(a_interior, a_boxes, a_domain)` described in section 2.5.1.
- `a_boxes` specifies the cell-centered layout of boxes.

**Output:**

- `a_exterior` contains the exterior boundary nodes of `a_boxes` in a `LayoutData` based on `a_boxes`. For each box in the layout of `a_boxes`, there is a `Vector` of `IntVectSets` that together contain the indices of the exterior boundary nodes of the box.

### 2.5.3 copyInteriorNodes function

The `copyInteriorNodes` function copies data from a source to a destination, but only from interior nodes of the source. Recall that the *interior* nodes are simply those that are not *exterior* nodes. This function is similar to `copyTo` but copies *valid* data only. See Figure 2.4 for an example.



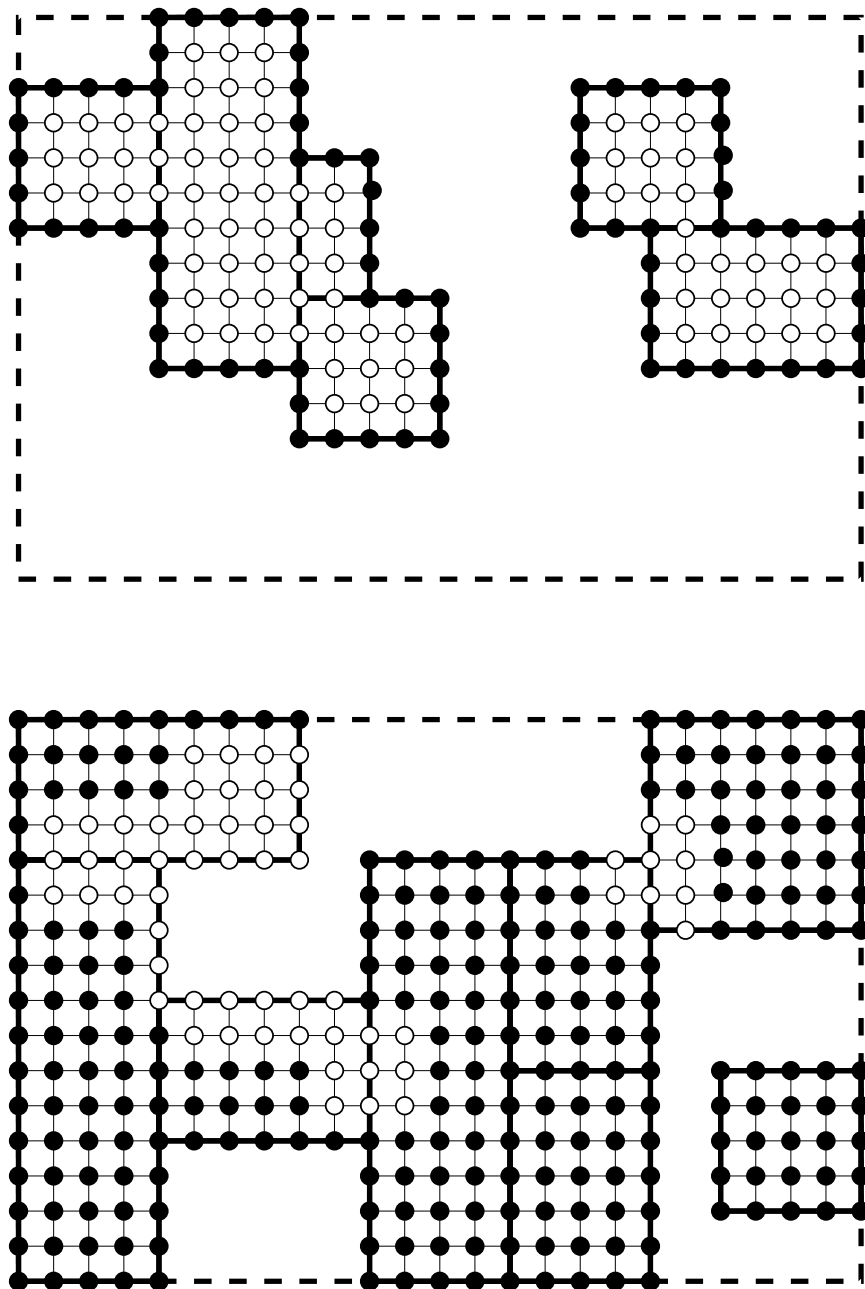


Figure 2.4: An example of a source and destination layout for `copyInteriorNodes`. The problem domain, delineated by dashed lines, is taken to be non-periodic.

Top: Hollow circles indicate the interior nodes of a source layout.

Bottom: Hollow circles indicate the nodes of a destination layout that are filled by `copyInteriorNodes` with data from the source layout above.

- `void copyInteriorNodes(`  
`LevelData<NodeFArrayBox>& a_dest,`  
`const LevelData<NodeFArrayBox>& a_src,`  
`const LayoutData< Vector<IntVectSet> >& a_IVSV)`

Copies data from the interior boundary nodes of `a_src` to `a_dest`.

**Inputs:**

- `a_src` contains the data to be copied at its interior nodes.
- `a_IVSV` is the object returned by  
`interiorBoundaryNodes(a_interior, a_dest.boxLayout(),`  
`a_src.boxLayout(), a_domain)`  
described in section 2.5.1.

**Output:**

- `a_dest`, at interior nodes of the layout of `a_src`, is replaced by data from `a_src`. Elsewhere, `a_dest` is unchanged.

## 2.5.4 zeroBoundaryNodes function

- `void zeroBoundaryNodes(`  
`BoxLayoutData<NodeFArrayBox>& a_dest,`  
`const LayoutData< Vector<IntVectSet> >& a_IVSV)`

Sets data to zero on specified nodes.

**Input:**

- `a_IVSV` is an object containing indices of nodes for each box, where data are to be set to zero. Usually it stores the indices of exterior boundary nodes of `a_dest`. These are obtained with the sequence of calls:

```
interiorBoundaryNodes(a_IVSVint, a_dest.boxLayout(), a_domain);
exteriorBoundaryNodes(a_IVSV, a_IVSVint, a_dest.boxLayout());
```

where `a_domain` is the cell-centered problem domain.

**Output:**

- `a_dest` is set to zero at nodes in `a_IVSV`. Elsewhere, `a_dest` is unchanged.

The underlying layouts of `a_dest` and `a_IVSV` must be the same.

# Chapter 3

## Interface for AMRNodeElliptic Solver

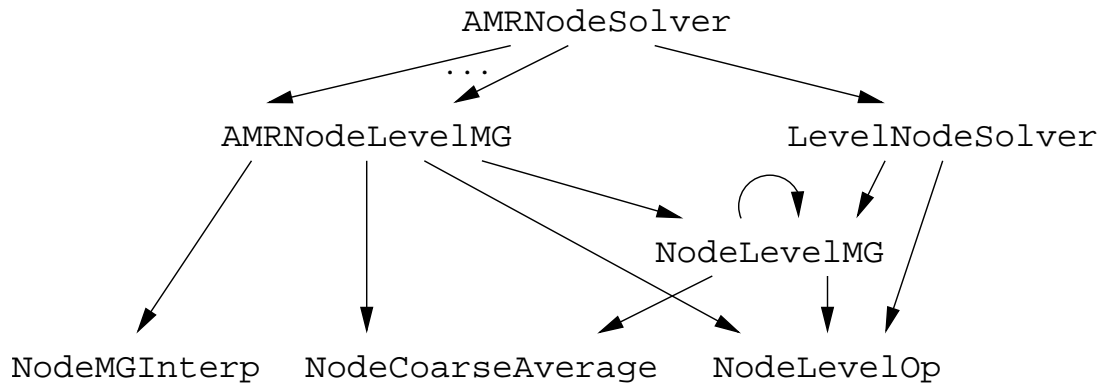


Figure 3.1: This chart shows the major classes of `AMRNodeElliptic` used by `AMRNodeSolver`. An arrow from one class to another indicates that an instance of the first class defines an instance of the second class. In each case, only one instance of the second class is defined, except that the `AMRNodeSolver` defines an instance of `AMRNodeLevelMG` for each level in the AMR hierarchy.

### 3.1 The class `AMRNodeSolver`

The `AMRNodeSolver` class solves an elliptic equation on an adaptive mesh refinement hierarchy of node-centered grids. The boundary conditions are obtained from a combination of interpolation from the next coarser level of grids, plus some set of physical boundary conditions on a rectangular domain.

The API is as follows.

- `void define(  
    const Vector<DisjointBoxLayout>& a_gridsLevel,  
    const Vector<ProblemDomain>& a_domainLevel,`

```

const Vector<Real>&          a_dxLevel,
const Vector<int>&          a_refRatio,
int                        a_numLevels,
int                        a_lBase,
const NodeLevelOp* const  a_opin,
int                        a_minLength = 1)

```

```

• void define(
  const Vector<DisjointBoxLayout>& a_gridsLevel,
  const Vector<Box>&                a_domainLevel,
  const Vector<Real>&              a_dxLevel,
  const Vector<int>&              a_refRatio,
  int                              a_numLevels,
  int                              a_lBase,
  const NodeLevelOp* const        a_opin,
  int                              a_minLength = 1)

```

Sets up the internal state of an AMRNodeSolver object with information about the grid hierarchy and the operator.

**Inputs:**

- `a_gridsLevel` specifies the layout of the cell-centered grids on all levels. Vector index corresponds to level number.
- `a_domainLevel` specifies the entire cell-centered problem domain at all levels of resolution. Vector index corresponds to level number.
- `a_dxLevel` specifies the mesh spacing at all levels. Vector index corresponds to level number.
- `a_refRatio` specifies the refinement ratios between adjacent levels. `a_refRatio[ilev]` is the refinement ratio between levels `ilev` and `ilev+1`. `a_refRatio[ilev]` must be a power of two, with `a_refRatio[ilev] ≥ 1`.
- `a_numLevels` is the number of AMR levels in the calculation. The length of the Vectors `a_gridsLevel`, `a_domainLevel` and `a_dxLevel` must be at least `a_numLevels`, and the length of the Vector `a_refRatio` must be at least `a_numLevels-1`.
- `a_lBase` is the index of the coarsest level on which the solution is to be computed.
- `a_opin` is a pointer to the `NodeLevelOp` to use in solving.
- `a_minLength` is the minimum length of the maximally coarsened box in `LevelNodeSolver`, or 0 if there is to be no coarsening. If `a_minLength ≥ 1` then the multigrid solver will coarsen boxes only so far as to maintain the lengths of all boxes as multiples of `a_minLength`.

- `void solveAMR(`  
`Vector<LevelData<NodeFArrayBox>*>&        a_phiLevel,`  
`const Vector<LevelData<NodeFArrayBox>*>& a_rhsLevel)`  
 Solves the elliptic equation over the hierarchy of levels `m_lBase` ... `m_finestLevel` where `m_finestLevel = m_numLevels-1`. If `m_lBase > 0`, then the data at level `m_lBase-1` is used to interpolate boundary conditions at boundary cells that are not adjacent to the domain boundary. Solves to tolerance `m_tolerance`.  
**Inputs:**
  - `a_phiLevel` contains pointers to the current guess at the solution values for levels (`lMin = max(m_lBase-1, 0)`) .. `m_finestLevel`. Index in the Vector corresponds to level number. These values are updated in place.
  - `a_rhsLevel` contains pointers to values of the right-hand side for levels `m_lBase` .. `m_finestLevel`.**Outputs:**
  - `a_phiLevel` contains pointers to values of the updated solution at the same levels.
- `void AMRVCycleMG(`  
`Vector<LevelData<NodeFArrayBox>*>&        a_phiLevel,`  
`const Vector<LevelData<NodeFArrayBox>*>& a_rhsLevel)`  
 Does one relaxation V-cycle using an AMR multigrid solver.  
**Inputs:**
  - `a_phiLevel` contains pointers to current guess at values of the solution at levels `m_lBase` .. `m_finestLevel`. Vector index corresponds to level number.
  - `a_rhsLevel` contains pointers to values of the right-hand side on the same levels as `a_phiLevel`.**Outputs:**
  - `a_phiLevel` contains pointers to values of the updated solution.
- `Real computeResidualNorm(int a_normNtype)`  
 Returns the norm of the multilevel residual on levels `m_lBase` to `m_finestLevel`, where the residual at level `ilev` has been stored in `m_amrmgLevel[ilev]->m_resid`. The residual must have been computed already before this function is called.  
**Inputs:**
  - `a_normType` is the type of norm: 0 for max norm, or  $p > 0$  for  $L_p$  norm.

- `void applyAMROperator(
 LevelData<NodeFArrayBox>& a_lofPhi,
 Vector<LevelData<NodeFArrayBox>*>& a_phiLev,
 int a_ilev)`

Applies the multilevel AMR operator to the data on a level.

**Inputs:**

- `a_phiLev` contains pointers to current guess at values of the solution at levels `m_lBase .. m_fineestLevel`. Vector index corresponds to level number.
- `a_ilev` is level on which operator is to be applied. The result depends on `a_phiLev[a_ilev]`, as well as `a_phiLev[a_ilev-1]` and `a_phiLev[a_ilev+1]` if they are defined.

**Outputs:**

- `a_lofPhi` contains value of operator applied on valid region of level `a_ilev`.

- `void computeAMRResidual(
 LevelData<NodeFArrayBox>& a_res,
 Vector<LevelData<NodeFArrayBox>*>& a_phiLevel,
 const Vector<LevelData<NodeFArrayBox>*>& a_rhsLevel,
 int a_ilev)`

Computes the residual on a level using the multilevel operator.

**Inputs:**

- `a_phiLevel` contains pointers to current guess at values of the solution at levels `m_lBase .. m_fineestLevel`. Vector index corresponds to level number.
- `a_rhsLevel` contains pointers to the right-hand side at the same levels as `a_phiLevel`.
- `a_ilev` is level on which the residual is to be computed. The result depends on `a_rhsLevel[a_ilev]` and `a_phiLevel[a_ilev]`, as well as `a_phiLevel[a_ilev-1]` and `a_phiLevel[a_ilev+1]` if they are defined.

**Outputs:**

- `a_res` contains residual on valid region of level `a_ilev`. This is also stored in `m_amrmgLevel[ilev]->m_resid`.

- `void setTolerance(Real a_tolerance)`

Sets the tolerance in the AMR solver: iterations end when the norm of the residual is less than `a_tolerance` times the initial residual. Default is `1.0e-10`.

- `void setOperatorTolerance(Real a_operatorTolerance)`  
Sets the “operator tolerance” of the AMR solver: iterations end if the ratio of the new residual to the old residual exceeds  $1 - a\_operatorTolerance$  and at least `m_minIter` iterations have been performed. Default is  $1.0e-5$ .
- `void setBottomTolerance(Real a_tolerance)`  
Sets the tolerance in the bottom solver, `LevelNodeSolver m_levelSolver`.
- `void setMaxIter(int a_maxIter)`  
Sets the maximum number of iterations in the AMR solver. Default is 42.
- `void setMinIter(int a_minIter)`  
Sets the minimum number of iterations in the AMR solver before it will terminate due to lack of convergence. Default is 5.
- `void setBottomMaxIter(Real a_maxIter)`  
Sets the maximum number of iterations in the bottom solver, `LevelNodeSolver::levelSolve` or `LevelNodeSolver::levelSolveH`. Default is 33.
- `void setNumSmoothDown(int a_numSmoothDown)`  
Sets the number of iterations of smoothing for the downward part of the V-cycle in `NodeLevelMG::mgRelax`. Default is 4.
- `void setNumSmoothUp(int a_numSmoothUp)`  
Sets the number of iterations of smoothing for the upward part of the V-cycle in `NodeLevelMG::mgRelax`. Default is 4.
- `void setNumBottomGSRB(int a_numBottomGSRB)`  
Sets the number of iterations of smoothing at the bottom of the V-cycle in `NodeLevelMG::mgRelax`. Default is 16.
- `void setBottomSmoothing(bool a_doBottomSmooth)`  
Sets whether the smoother is applied at the bottom of the V-cycle in `NodeLevelMG::mgRelax`. Default is true.

### Principal data members

- `Vector<AMRNodeLevelMG*> m_amrmgLevel` is a vector of length `m_numLevels` containing pointers to `AMRNodeLevelMG` objects, each of which manages the data and operations on one level.

- `LevelNodeSolver m_levelSolver` solves elliptic equations on level `m_lBase` using multigrid.
- `Vector<DisjointBoxLayout> m_gridsLevel`, `Vector<Box> m_domainLevel`, `Vector<Real> m_dxLevel`, `Vector<int> m_refRatio`, `int m_numLevels`, `int m_lBase` are set to the arguments `a_gridsLevel`, `a_domainLevel`, `a_dxLevel`, `a_refRatio`, `a_numLevels`, `a_lBase` in the constructor.

### 3.1.1 Internal class `AMRNodeLevelMG`

`AMRNodeLevelMG` manages the data and operations for `AMRNodeSolver` on one level. `AMRNodeSolver` defines a vector of `AMRNodeLevelMG` objects, one for each level. This class should be considered internal to `AMRNodeSolver` and should not be considered part of the Chombo API.

These functions are called from `AMRNodeSolver`. In a multigrid V-cycle, `AMRNodeSolver::AMRVCycleMG` calls `downSweep` at each level from the finest down to one above the base level, solves at the base level with class `LevelNodeSolver`, and then calls `upSweep` at each level from one above the base up to the finest level.

- `void define(`  
`const AMRNodeSolver* const a_parent,`  
`int                          a_level,`  
`const NodeLevelOp* const   a_opin)`

Sets up the internal state of an `AMRNodeLevelMG` object.

**Inputs:**

- `a_parent` is a pointer to the `AMRNodeSolver` object that called this constructor.
- `a_level` is the level for which this object manages data and operations.
- `a_opin` is a pointer to the `NodeLevelOp` to use in solving.

- `void applyAMROperator(`  
`LevelData<NodeFArrayBox>&          a_Lofphi,`  
`Vector<LevelData<NodeFArrayBox>*>& a_phiLevel)`

Calculates the multilevel AMR operator to the data on this level. This operator uses inhomogeneous coarse-fine boundary conditions and inhomogeneous physical domain boundary conditions.

**Inputs:**

- `a_phiLevel` contains pointers to current guess at values of the solution. Vector index corresponds to level number. The result depends on `a_phiLev[m_level]`, as well as `a_phiLev[m_level-1]` and `a_phiLev[m_level+1]` if they are defined.



### Outputs:

- `a_lofPhi` contains value of operator applied on valid region of level `m_level`.

- `void computeAMRResidual(`  
    `Vector<LevelData<NodeFArrayBox>*>& a_phiLevel,`  
    `const Vector<LevelData<NodeFArrayBox>*>& a_rhsLevel)`

Computes the residual on this level using the multilevel operator. The result is stored in local data `m_resid`.

### Inputs:

- `a_phiLevel` contains pointers to current guess at values of the solution. Vector index corresponds to level number. The result depends on `*a_phiLevel[m_level]`, as well as `*a_phiLevel[m_level-1]` and `*a_phiLevel[m_level+1]` if they are defined.
- `a_rhsLevel` contains pointers to the right-hand side at the same levels as `a_phiLevel`. The result depends on `*a_rhsLevel[m_level]`.

- `void downSweep(`  
    `Vector<LevelData<NodeFArrayBox>*>& a_phiLevel,`  
    `const Vector<LevelData<NodeFArrayBox>*>& a_rhsLevel)`

Sweeps down a multigrid V-cycle from this level to next coarser level. This function calculates the correction `m_corr` and updates the solution at this level with it. It also overwrites the residual at the next coarser level, if any, with the averaged residual at this level.

### Inputs:

- `a_phiLevel` contains pointers to current guess at values of the solution. Vector index corresponds to level number.
- `a_rhsLevel` contains pointers to the right-hand side at the same levels as `a_phiLevel`.

### Outputs:

- `a_phiLevel` contains pointers to updated solution.

- `void upSweep(`  
    `Vector<LevelData<NodeFArrayBox>*>& a_phiLevel,`  
    `const Vector<LevelData<NodeFArrayBox>*>& a_rhsLevel)`

Sweeps up a multigrid V-cycle from the next coarser level to this level. This function interpolates the correction `m_corr` from the next coarser level, uses it to finish the calculation of residual `m_resid`, and updates the solution at this level.

### Inputs:

- `a_phiLevel` contains pointers to current guess at values of the solution. Vector index corresponds to level number.
- `a_rhsLevel` contains pointers to the right-hand side at the same levels as `a_phiLevel`.

**Outputs:**

- `a_phiLevel` contains pointers to updated solution.
- Real `computeResidualNorm(int a_normNtype)`

Returns the norm of internal data `m_resid`.

**Inputs:**

- `a_normType` is the type of norm: 0 for max norm, or  $p > 0$  for  $L_p$  norm.

**Principal data members**

- `LevelData<NodeFArrayBox> m_resid` is the residual calculated in `computeAMRResidual`.
- `LevelData<NodeFArrayBox> m_corr` is the correction at this level.
- `int m_level` is the level for which this object manages data and operations, set to `a_level` argument in constructor.
- `NodeLevelOp* m_levelOpPtr` is the pointer to the level operator, set to `a_opin` argument in constructor.
- `NodeMGInterp m_mginterp` is used for interpolating the correction from the next coarser level to this level in `upSweep`.
- `NodeCoarseAverage m_averageOp` is used for averaging the residual down to the next coarser level in `downSweep`.

**3.1.2 Internal class NodeLevelMG**

The `NodeLevelMG` class is a multigrid solver on a level. It is used by the classes `LevelNodeSolver` and `AMRNodeLevelMG` (internal to `AMRNodeSolver`). This class should be considered internal to `AMRNodeSolver` and `LevelNodeSolver`, and should not be considered part of the Chombo API.

- `void define(`  
`const DisjointBoxLayout& a_grids,`  
`const DisjointBoxLayout* a_gridsCoarsePtr,`  
`const ProblemDomain& a_domain,`

```

Real          a_dx,
int           a_refToCoarse,
const NodeLevelOp* const a_opin,
int           a_nCoarserLevels)

```

- void define(

```

const DisjointBoxLayout& a_grids,
const DisjointBoxLayout* a_gridsCoarsePtr,
const Box&                a_domain,
Real                      a_dx,
int                       a_refToCoarse,
const NodeLevelOp* const a_opin,
int                       a_nCoarserLevels)

```

Defines the internal state of the NodeLevelMG object and allocates space for the residual. The arguments are the same as for the LevelNodeSolver constructor, but instead of `a_minLength` we have `a_nCoarserLevels`. This argument represents the number of coarser-level NodeLevelMG objects to be defined recursively. Namely,

- when defined from LevelNodeSolver,
  - \* if `a_minLength` is 0 then `a_nCoarserLevels` is also 0;
  - \* if `a_minLength`  $\geq 1$  then `a_nCoarserLevels` is the exponent of the largest power of 2 that divides the lengths of every grid at this level divided by `a_minLength`. Thus after `a_nCoarserLevels` coarsenings by 2, the lengths of all grids will still be divisible by `a_minLength`.
- when defined from AMRNodeLevelMG,
  - \* if `a_minLength` is 0 or `a_level` is 0 then `a_nCoarserLevels` is also 0;
  - \* if `a_minLength`  $\geq 1$  and `a_level`  $\geq 1$  then `a_nCoarserLevels` =  $\log_2(r) - 1$  where  $r$  is the refinement ratio to the next coarser level.

- void mgRelax(

```

LevelData<NodeFArrayBox>&    a_phi,
const LevelData<NodeFArrayBox>& a_rhs,
bool                        a_bottomsolveflag)

```

Invokes a relaxation step, updating `a_phi` with right-hand side `a_rhs`. It is assumed that the problem is in residual-correction form. In particular, only the homogeneous form of the physical and coarse-fine boundary conditions need be invoked. `mgRelax` calls itself recursively on the coarsened grids.

## Principal data members

- LevelData<NodeFArrayBox> `m_resid` is the residual at this level.

- `LevelData<NodeFArrayBox> m_crseResid` is the residual on the grids of this level coarsened by 2.
- `LevelData<NodeFArrayBox> m_crseCorr` is the correction on the grids of this level coarsened by 2.
- `NodeLevelOp* m_levelOpPtr` is the pointer to the level operator, set to `a_opin` argument in constructor.
- `NodeCoarseAverage m_averageOp` is used for averaging the residual down to the residual on the coarsened grids of this level, in `mgRelax`.

## 3.2 The class `LevelNodeSolver`

The `LevelNodeSolver` class solves elliptic equations on a level using the multigrid method. It is used by `AMRNodeSolver` to solve at the base level.

The API is as follows.

- `void define(`  
`const DisjointBoxLayout& a_grids,`  
`const DisjointBoxLayout* a_gridsCoarsePtr,`  
`const ProblemDomain&    a_domain,`  
`Real                    a_dx,`  
`int                    a_refToCoarse,`  
`const NodeLevelOp* const a_opin,`  
`int                    a_minLength = 1)`
- `void define(`  
`const DisjointBoxLayout& a_grids,`  
`const DisjointBoxLayout* a_gridsCoarsePtr,`  
`const Box&              a_domain,`  
`Real                    a_dx,`  
`int                    a_refToCoarse,`  
`const NodeLevelOp* const a_opin,`  
`int                    a_minLength = 1)`

Sets up the internal state of a `LevelNodeSolver` object.

### Inputs:

- `a_grids` specifies the layout of the cell-centered grids at this level.
- `a_gridsCoarsePtr` is a pointer to the layout of the cell-centered grids at the next coarser level, or `NULL` if there is no coarser level.
- `a_domain` specifies the cell-centered problem domain on this level.

- a\_dx is the mesh spacing at this level.
- a\_refToCoarse is the refinement ratio between this level and the next coarser level. Ignored if there is no coarser level.
- a\_opin is a pointer to the NodeLevelOp to use in solving.
- a\_minLength is the minimum length of the maximally coarsened box, or 0 if there is to be no coarsening. If  $a\_minLength \geq 1$  then the multigrid solver will coarsen boxes only so far as to maintain the lengths of all boxes as multiples of a\_minLength.

- void levelSolve(
 

```

LevelData<NodeFArrayBox>&      a_phi,
const LevelData<NodeFArrayBox>* a_phiCoarse,
const LevelData<NodeFArrayBox>& a_rhs,
bool                          a_initializePhiToZero = true)

```

Does a level solve on this level using multigrid and inhomogeneous boundary conditions at coarse/fine interfaces.

**Inputs:**

- a\_phi is initial guess at solution at this level.
- a\_phiCoarse is a pointer to solution at next coarser level, or NULL if there is no coarser level.
- a\_rhs contains the right-hand side at this level.
- a\_initializePhiToZero tells whether to initialize the solution to zero.

**Output:**

- a\_phi is updated solution at this level.

- void levelSolveH(
 

```

LevelData<NodeFArrayBox>&      a_phi,
const LevelData<NodeFArrayBox>& a_rhs,
bool                          a_initializePhiToZero = true)

```

Does a level solve on this level using multigrid and homogeneous boundary conditions at coarse/fine interfaces.

**Inputs:**

- a\_phi is initial guess at solution at this level.
- a\_rhs contains the right-hand side at this level.
- a\_initializePhiToZero tells whether to initialize the solution to zero.

**Output:**

- `a_phi` is updated solution at this level.
- `void setTolerance(Real a_tolerance)`  
Sets the tolerance in the level solver: iterations end when the norm of the residual is less than `a_tolerance` times the initial residual. Default is  $1.0e-10$ .
- `void setOperatorTolerance(Real a_operatorTolerance)`  
Sets the “operator tolerance” of the solver: iterations end if the ratio of the new residual to the old residual exceeds  $1 - a\_operatorTolerance$  and at least `m_minIter` iterations have been performed. Default is  $1.0e-4$ .
- `void setMaxIter(int a_maxIter)`  
Sets the maximum number of relaxation iterations. Default is 33.
- `void setMinIter(int a_minIter)`  
Sets the minimum number of iterations in the solver before it will terminate due to lack of convergence. Default is 4.
- `void setnumSmoothDown(int a_numSmoothDown)`  
Sets the number of iterations of smoothing for the downward part of the V-cycle in `m_levelMG.mgRelax`. Default is 4.
- `void setnumSmoothUp(int a_numSmoothUp)`  
Sets the number of iterations of smoothing for the upward part of the V-cycle in `m_levelMG.mgRelax`. Default is 4.
- `void setnumBottomGSRB(int a_numBottomGSRB)`  
Sets the number of iterations of smoothing at the bottom of the V-cycle in `m_levelMG.mgRelax`. Default is 16.
- `void setBottomSmoothing(bool a_bottomSolveFlag)`  
Sets whether the smoother is applied at the bottom of the V-cycle in `m_levelMG.mgRelax`. Default is true.

### Principal data members

- `NodeLevelMG m_levelMG` is a multigrid level solver object to relax on this level.
- `NodeLevelOp* m_levelOpPtr` is a pointer to the `NodeLevelOp` to use in solving.
- `LevelData<NodeFArrayBox> m_resid` is the residual in the level solve.
- `LevelData<NodeFArrayBox> m_corr` is the correction in the level solve.

### 3.3 The NodeLevelOp interface

NodeLevelOp is a pure base class to encapsulate level operations for node-centered elliptic solvers.

- virtual NodeLevelOp\* new\_levelop() = 0  
Returns a pointer to a new instance of NodeLevelOp of the same type. This gets around the “no virtual constructor” rule.
- virtual void define(  
    const DisjointBoxLayout& a\_grids,  
    const DisjointBoxLayout\* a\_gridsCoarsePtr,  
    Real a\_dx,  
    int a\_refToCoarse,  
    const ProblemDomain& a\_domain,  
    bool a\_homogeneousOnly = false,  
    int a\_ncomp = 1) = 0
- virtual void define(  
    const DisjointBoxLayout& a\_grids,  
    const DisjointBoxLayout\* a\_gridsCoarsePtr,  
    Real a\_dx,  
    int a\_refToCoarse,  
    const Box& a\_domain,  
    bool a\_homogeneousOnly = false,  
    int a\_ncomp = 1) = 0

Full define function. Makes all coarse/fine information and sets internal variables.

**Inputs:**

- a\_grids specifies the layout of the cell-centered grids at this level.
- a\_gridsCoarsePtr is a pointer to the layout of the cell-centered grids at the next coarser level, or NULL if there is no coarser level.
- a\_domain specifies the cell-centered problem domain on this level.
- a\_dx is the mesh spacing at this level.
- a\_refToCoarse is the refinement ratio between this level and the next coarser level. Ignored if there is no coarser level.
- a\_homogeneousOnly specifies whether all coarse/fine interpolation is homogeneous.
- a\_ncomp is the number of components in the operator argument.

- virtual void define(
 

```

      const NodeLevelOp* a_opfine,
      int a_refToFine) = 0
      
```

Full define function. Makes all coarse/fine information and sets internal variables from finer NodeLevelOp. Any NodeLevelOp defined with this function is not able to execute inhomogeneous boundary conditions at the coarse/fine interface.

**Inputs:**

- a\_opfine is the finer-level operator.
- a\_refToFine is the refinement ratio between this level and the next finer level.

- virtual void CFInterp(
 

```

      LevelData<NodeFArrayBox>& a_phi,
      const LevelData<NodeFArrayBox>& a_phiCoarse,
      bool a_inhomogeneous) = 0
      
```

Fills the nodes on the coarse/fine interface with interpolated data from the coarser level.

**Inputs:**

- a\_phi is the solution on the current level.
- a\_phiCoarse is the solution at the next coarser level.
- a\_inhomogeneous specifies whether the physical boundary condition is inhomogeneous.

- virtual void homogeneousCFInterp(
 

```

      LevelData<NodeFArrayBox>& a_phi) = 0
      
```

Zeros out the nodes that lie on the interface with the coarser level.

**Input:**

- a\_phi is the solution on this level.

**Output:**

- a\_phi is the solution on this level with interface nodes set to zero.

- virtual void smooth(
 

```

      LevelData<NodeFArrayBox>& a_phi,
      const LevelData<NodeFArrayBox>& a_rhs) = 0
      
```

Smoother. This smooths the solution (in the multigrid sense) on a level. It assumes that the problem has already been put into residual-correction form, so that coarse/fine boundary conditions are homogeneous.

**Inputs:**



- a\_phi is the solution on this level.
- a\_rhs is the right-hand side on this level.

**Output:**

- a\_phi is the smoothed solution on this level.

- virtual void applyOpH(
   
LevelData<NodeFArrayBox>& a\_LofPhi,
   
LevelData<NodeFArrayBox>& a\_phi) = 0
- virtual void applyOpI(
   
LevelData<NodeFArrayBox>& a\_LofPhi,
   
LevelData<NodeFArrayBox>& a\_phi,
   
const LevelData<NodeFArrayBox>\* a\_phiCoarsePtr) = 0
- virtual void applyOpHcfIphys(
   
LevelData<NodeFArrayBox>& a\_LofPhi,
   
LevelData<NodeFArrayBox>& a\_phi) = 0
- virtual void applyOpIcfHphys(
   
LevelData<NodeFArrayBox>& a\_LofPhi,
   
LevelData<NodeFArrayBox>& a\_phi,
   
const LevelData<NodeFArrayBox>\* a\_phiCoarsePtr) = 0

Evaluate the operator. The four functions differ in whether the boundary conditions are homogeneous or inhomogeneous on the coarse/fine interface and on the physical boundary:

function	coarse/fine interface	physical boundary
applyOpH	homogeneous	homogeneous
applyOpI	inhomogeneous	inhomogeneous
applyOpHcfIphys	homogeneous	inhomogeneous
applyOpIcfHphys	inhomogeneous	homogeneous

**Inputs:**

- a\_phi is the solution on this level.
- a\_phiCoarsePtr is a pointer to the solution at the next coarser level, required for inhomogeneous coarse/fine boundary conditions.

**Output:**

- a\_LofPhi is the result of the operator on this level.

- virtual void residualH(
 

```

      LevelData<NodeFArrayBox>&      a_resid,
      LevelData<NodeFArrayBox>&      a_phi,
      const LevelData<NodeFArrayBox>& a_rhs) = 0
      
```
- virtual void residualI(
 

```

      LevelData<NodeFArrayBox>&      a_resid,
      LevelData<NodeFArrayBox>&      a_phi,
      const LevelData<NodeFArrayBox>* a_phiCoarsePtr,
      const LevelData<NodeFArrayBox>& a_rhs) = 0
      
```
- virtual void residualHcfIphys(
 

```

      LevelData<NodeFArrayBox>&      a_resid,
      LevelData<NodeFArrayBox>&      a_phi,
      const LevelData<NodeFArrayBox>& a_rhs) = 0
      
```
- virtual void residualIcfHphys(
 

```

      LevelData<NodeFArrayBox>&      a_resid,
      LevelData<NodeFArrayBox>&      a_phi,
      const LevelData<NodeFArrayBox>* a_phiCoarsePtr,
      const LevelData<NodeFArrayBox>& a_rhs) = 0
      
```

Calculate the residual of the operator. If  $L$  is the operator,  $\varphi$  is the solution to which the operator is applied, and  $\rho$  is the right-hand side, then the residual is simply  $\rho - L(\varphi)$ . The four functions differ in whether the boundary conditions are homogeneous or inhomogeneous on the coarse/fine interface and on the physical boundary:

function	coarse/fine interface	physical boundary
residualH	homogeneous	homogeneous
residualI	inhomogeneous	inhomogeneous
residualHcfIphys	homogeneous	inhomogeneous
residualIcfHphys	inhomogeneous	homogeneous

### Inputs:

- a\_phi is the solution on this level.
- a\_phiCoarsePtr is a pointer to the solution at the next coarser level, required for inhomogeneous coarse/fine boundary conditions.
- a\_rhs is the right-hand side on this level.

### Output:

- a\_resid is the residual on this level.

- virtual void bottomSmoother(
 

```

          LevelData<NodeFArrayBox>&      a_phi,
          const LevelData<NodeFArrayBox>& a_rhs) = 0
      
```

Performs smoothing at the bottom level. This is used when it is not possible to coarsen the grid. Typically, the function is either a point relaxation or a conjugate-gradient type of method.

**Inputs:**

- a\_phi is the solution on this level.
- a\_rhs is the right-hand side on this level.

**Output:**

- a\_phi is the smoothed solution on this level.

- virtual void levelPreconditioner(
 

```

          LevelData<NodeFArrayBox>&      a_phihat,
          const LevelData<NodeFArrayBox>& a_rhshat) = 0
      
```

Applies preconditioner. In the notation of [BBC<sup>+</sup>94], if the preconditioner is  $M$ , which is an approximation to the operator, then this solves the related equation  $M\hat{\phi} = \hat{\rho}$ . In the AMR multigrid implementation, this function is generally most often used by the bottom solver.

**Input:**

- a\_rhshat is the right-hand side of the preconditioning.

**Output:**

- a\_phihat is the result of the preconditioning.

### 3.3.1 The NodeBaseBottomSmoother interface

NodeBaseBottomSmoother is a pure base class to encapsulate operations for smoothers used by node-centered elliptic solvers.

- virtual NodeBaseBottomSmoother\* new\_bottomSmoother() const = 0
 

Returns a pointer to a new instance of NodeBaseBottomSmoother of the same type.
- virtual void doBottomSmooth(
 

```

          LevelData<NodeFArrayBox>&      a_phi,
          const LevelData<NodeFArrayBox>& a_rhs,
          NodeLevelOp*                  a_levelop_ptr) = 0
      
```

Performs smoothing of  $L(\varphi) = \rho$ .

**Inputs:**

- `a_phi` is the solution on a level.
- `a_rhs` is the right-hand side on a level.
- `a_levelop_ptr` is a pointer to an instance of the operator.

**Output:**

- `a_phi` is the smoothed solution on the level.

Examples of smoothers are conjugate gradient and BiCGStab, described in [BBC<sup>+</sup>94]. These are implemented as the `NodeCGSmoother` and `NodeBiCGStabSmoother` classes, respectively, described in detail in [McC02].

### 3.3.2 The class `NodePoissonOp`

`NodePoissonOp` is a subclass of `NodeLevelOp` that includes operations for the discrete Laplacian operator. Here we describe only the functions and members of `NodePoissonOp` that are not described in the section on `NodeLevelOp`.

- `void setBottomSmoother(const NodeBaseBottomSmoother& a_bottomSmoother)`  
Sets the bottom smoother to be used in the `smooth` function. The default bottom smoother for `NodePoissonOp` is an object of class `NodeCGSmoother`.
- `void setInterpolationDegree(int a_interpolationDegree)`  
Sets the degree of interpolation to be used in `NodeQuadCFInterp`: 1 for linear in two dimensions or bilinear in three dimensions; 2 for quadratic in two dimensions or biquadratic in three dimensions. Default is 2. This function must be called before `define`. The interpolation degree cannot be changed later without calling `define` again.
- `void setDomainNodeBC(const DomainNodeBC& a_dombcIn)`  
Sets the boundary conditions of the physical domain.

## 3.4 The class `AMRNodeSolverAlt`

The `AMRNodeSolverAlt` class solves an elliptic equation on an adaptive mesh refinement hierarchy using an alternative algorithm to the AMR multigrid solver implemented in `AMRNodeSolver`. `AMRNodeSolverAlt` solves one level at a time, from coarsest to finest, interpolating from coarser to finer solutions as it proceeds. See Figure 3.2 for a chart of major classes used in `AMRNodeSolverAlt`.

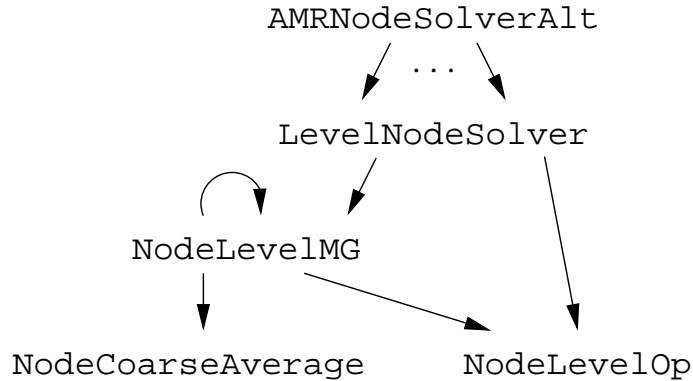


Figure 3.2: This chart shows the major classes of AMRNodeElliptic used by AMRNodeSolverAlt. An arrow from one class to another indicates that an instance of the first class defines an instance of the second class. In each case, only one instance of the second class is defined, except that the AMRNodeSolverAlt defines an instance of LevelNodeSolver for each level in the AMR hierarchy.

- void define(
 

const Vector<DisjointBoxLayout>&	a_gridsLevel,
const Vector<ProblemDomain>&	a_domainLevel,
const Vector<Real>&	a_dxLevel,
const Vector<int>&	a_refRatio,
int	a_numLevels,
int	a_lBase,
const NodeLevelOp* const	a_opin,
int	a_minLength = 1)
- void define(
 

const Vector<DisjointBoxLayout>&	a_gridsLevel,
const Vector<Box>&	a_domainLevel,
const Vector<Real>&	a_dxLevel,
const Vector<int>&	a_refRatio,
int	a_numLevels,
int	a_lBase,
const NodeLevelOp* const	a_opin,
int	a_minLength = 1)

Sets up the internal state of an AMRNodeSolverAlt object with information about the grid hierarchy and the operator. The arguments are the same as for the define function of AMRNodeSolver.

- void solveAMR(
 

Vector<LevelData<NodeFArrayBox>*>&	a_phiLevel,
------------------------------------	-------------

```
const Vector<LevelData<NodeFArrayBox>*>& a_rhsLevel)
```

Solves the elliptic equation over a hierarchy of levels. The arguments are the same as for the `solveAMR` function of `AMRNodeSolver`.

- `void setTolerance(Real a_tolerance)`  
Sets the tolerance in the one-level solvers: iterations at a level end when the norm of the residual is less than `a_tolerance` times the initial residual. Default is  $1.0e-10$ .
- `void setMaxIter(int a_maxIter)`  
sets the maximum number of iterations in the solvers at all levels. Default is set in `LevelNodeSolver`.
- `void setBottomSmoothing(bool a_doBottomSmooth)`  
sets whether the smoother is applied at the bottom of the V-cycle in the function `NodeLevelMG::mgRelax`. Default is `true`.

### Principal data members.

- `Vector<LevelNodeSolver*> m_levelSolver` is a vector of length `m_numLevels` containing pointers to `LevelNodeSolver` objects, each of which manages the data and operations on one level.
- `Vector<DisjointBoxLayout> m_gridsLevel`, `Vector<Box> m_domainLevel`, `Vector<Real> m_dxLevel`, `Vector<int> m_refRatio`, `int m_numLevels`, `int m_lBase` are set to the arguments `a_gridsLevel`, `a_domainLevel`, `a_dxLevel`, `a_refRatio`, `a_numLevels`, `a_lBase` in the constructor.

# Chapter 4

## C++ Classes for Two-Level Operators

### 4.1 The class NodeCoarseAverage

The NodeCoarseAverage class is used for replacing coarse-level data with an average of fine-level data in the classes NodeLevelMG and AMRNodeLevelMG.

- `void define(  
    const DisjointBoxLayout& a_gridsFine,  
    const DisjointBoxLayout& a_gridsCoarse,  
    int                        a_numcomps,  
    int                        a_refRatio,  
    const ProblemDomain&      a_domainFine)`
- `void define(  
    const DisjointBoxLayout& a_gridsFine,  
    const DisjointBoxLayout& a_gridsCoarse,  
    int                        a_numcomps,  
    int                        a_refRatio,  
    const Box&                 a_domainFine)`

Defines the internal state of the NodeCoarseAverage object.

**Inputs:**

- `a_gridsFine` specifies the layout of the cell-centered grids at the finer level.
- `a_gridsCoarse` specifies the layout of the cell-centered grids at the coarser level.
- `a_numcomps` is the number of components in the data.
- `a_refRatio` is the refinement ratio between the two levels.
- `a_domainFine` specifies the cell-centered problem domain on the finer level.

- `void define(`  
`const DisjointBoxLayout& a_gridsCoarse,`  
`int                    a_numcomps,`  
`int                    a_refRatio,`  
`const ProblemDomain&   a_domainFine)`
- `void define(`  
`const DisjointBoxLayout& a_gridsCoarse,`  
`int                    a_numcomps,`  
`int                    a_refRatio,`  
`const Box&              a_domainFine)`

Defines the internal state of the `NodeCoarseAverage` object. The arguments are the same as that of the longer `define` function above, except that `a_gridsFine` is not present; the grids at the finer level are taken to be refinements of the grids at the coarser level.

- `void averageToCoarse(`  
`LevelData<NodeFArrayBox>& a_coarse,`  
`LevelData<NodeFArrayBox>& a_fine)`

Replaces coarse-level data with an average of nearby finer-level cells' data, on the interior of the coarsened finer-level domain. Elsewhere, the coarse-level data is unchanged.

**Input:**

- `a_fine` is finer-level data.

**Output:**

- `a_coarse` is coarser-level data with interior nodes replaced by average of finer-level data.

## 4.2 The class `NodeMGInterp`

The `NodeMGInterp` class is used for interpolating data from a coarser to a finer AMR level in the class `AMRNodeLevelMG`.

- `void define(`  
`const DisjointBoxLayout& a_grids,`  
`int                    a_numcomps,`  
`int                    a_refRatio,`  
`const ProblemDomain&   a_domain)`



- `void define(`  
`const DisjointBoxLayout& a_grids,`  
`int                    a_numcomps,`  
`int                    a_refRatio,`  
`const Box&              a_domain)`

Defines the internal state of the `NodeMGInterp` object.

**Inputs:**

- `a_grids` specifies the layout of the cell-centered grids at the finer level.
- `a_numcomps` is the number of components in the data.
- `a_refRatio` is the refinement ratio between the two levels.
- `a_domain` specifies the cell-centered problem domain on the finer level.

- `void interpToFine(`  
`LevelData<NodeFArrayBox>&      a_fine,`  
`const LevelData<NodeFArrayBox>& a_coarse)`  
`bool                            a_sameGrids = false)`

Modifies the finer-level data by adding interpolated values of the coarser-level data.

**Input:**

- `a_fine` is original finer-level data.
- `a_coarse` is coarser-level data.
- `a_sameGrids` specifies whether the finer-level grids are refinements of the coarser-level grids.

**Output:**

- `a_fine` is updated finer-level data.

## 4.3 The class `NodeQuadCFInterp`

The `NodeQuadCFInterp` class interpolates data from a coarse level to a fine level on coarse/fine interfaces.

- `void define(`  
`const DisjointBoxLayout&          a_grids,`  
`Real                              a_dx,`  
`const ProblemDomain&              a_domain,`  
`const LayoutData<NodeCFIVS>* const a_loCFIVS,`  
`const LayoutData<NodeCFIVS>* const a_hiCFIVS,`  
`int                               a_refToCoarse,`

```

int                                     a_interpolationDegree = 2,
int                                     a_ncomp = 1)

```

- void define(

```

const DisjointBoxLayout&             a_grids,
Real                                 a_dx,
const Box&                           a_domain,
const LayoutData<NodeCFIVS>* const a_loCFIVS,
const LayoutData<NodeCFIVS>* const a_hiCFIVS,
int                                   a_refToCoarse,
int                                   a_interpolationDegree = 2,
int                                   a_ncomp = 1)

```

Defines the internal state of the NodeQuadCFInterp object.

**Inputs:**

- a\_grids specifies the layout of the cell-centered grids at the finer level.
- a\_dx is the mesh spacing at the finer level.
- a\_domain specifies the cell-centered problem domain on the finer level.
- a\_loCFIVS and a\_hiCFIVS are pointers to arrays of length SpaceDim of LayoutData<NodeCFIVS>, each of which holds information about nodes on the coarse/fine interface for each finer-level box. The underlying layout in every case is a\_grids. The arrays of LayoutData<NodeCFIVS> are indexed by dimension, with a\_loCFIVS for faces on the low sides and a\_hiCFIVS for faces on the high sides.
- a\_refToCoarse is the refinement ratio between the two levels. It must be a power of 2.
- a\_interpolationDegree is the degree of interpolation. It is 1 if interpolation is linear in two dimensions or bilinear in three dimensions. It is 2 if interpolation is quadratic in two dimensions or biquadratic in three dimensions.
- a\_ncomp is the number of components in the data.

- void coarseFineInterp(

```

LevelData<NodeFArrayBox>&           a_phiFine,
const LevelData<NodeFArrayBox>& a_phiCoarse,
bool                                 a_inhomogeneous)

```

Fills the nodes of the fine level of the coarse/fine interfaces with interpolated data from the coarser level.

**Inputs:**

- a\_phiCoarse is coarser-level data.

- `a_inhomogeneous` specifies whether the physical boundary condition is inhomogeneous.

**Output:**

- `a_phiFine` is the finer-level data, modified at nodes on interfaces with the coarser level.
- `void setDomainNodeBC(const DomainNodeBC& a_dombcIn)`  
Sets the boundary conditions of the physical domain. This is required if the refinement ratio is more than 2, because then the boundary conditions need to be set on the intermediate levels between successive interpolations by refinement ratio of 2.

## 4.4 The class NodeCFIVS

The NodeCFIVS class determines the set of nodes that lie on the coarse/fine interface, for a particular face of a particular box at the finer level.

- `void define(`  
`const ProblemDomain&    a_domain,`  
`const Box&              a_box,`  
`const DisjointBoxLayout& a_levelBoxes,`  
`int                      a_idir,`  
`Side::LoHiSide          a_hiorlo)`
- `void define(`  
`const Box&              a_domain,`  
`const Box&              a_box,`  
`const DisjointBoxLayout& a_levelBoxes,`  
`int                      a_idir,`  
`Side::LoHiSide          a_hiorlo)`

Defines the internal state of the NodeCFIVS object.

**Inputs:**

- `a_domain` specifies the cell-centered problem domain on the finer level.
- `a_box` specifies the finer-level box.
- `a_levelBoxes` specifies the layout of all cell-centered boxes at the finer level.
- `a_idir` and `a_hiorlo` specify the particular face of `a_box`.
- `const IntVectSet& getFineIVS() const`  
Returns the indices of the finer-level nodes on this face that also lie on the interface with the coarser level. The IntVectSet excludes indices of nodes that lie on the

boundary of the problem domain, and nodes that are on faces of any other boxes in `a_levelBoxes` on the side of `a_box` specified by `a_idir` and `a_hiorlo`. See Figure 4.1.

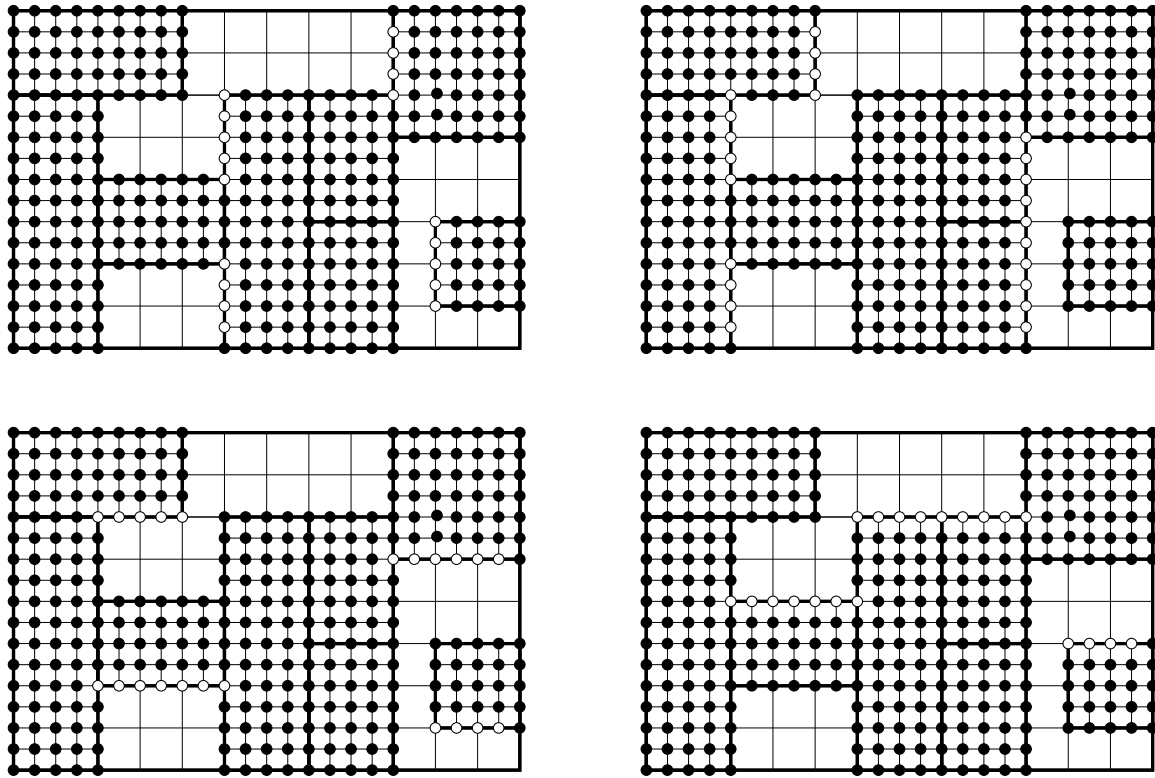


Figure 4.1: This figure indicates, with hollow circles, all nodes with indices returned by `getFineIVS` over all settings of `a_box` in a two-dimensional layout over a non-periodic domain.

Top: `a_idir = 0`, and `a_hiorlo = (left) Side::Lo` or `(right) Side::Hi`.

Bottom: `a_idir = 1`, and `a_hiorlo = (left) Side::Lo` or `(right) Side::Hi`.

# Chapter 5

## Boundary Conditions

Boundary conditions are enforced with the class `DomainNodeBC`, which contains a pointer to an instance of the class `FaceNodeBC` for each boundary face. The user adds a `FaceNodeBC`-derived class for each face to enforce particular boundary conditions.

The `AMRNodeElliptic` package allows boundary conditions that are expressed as

$$A \frac{\partial \varphi}{\partial n} + B \varphi = C \quad (5.1)$$

for each node lying on the boundary, where  $A$ ,  $B$ , and  $C$  may vary over the boundary. The class derived from `FaceNodeBC` provides a function to fill  $A$ ,  $B$ , and  $C$  on a boundary face. `DomainNodeBC` calls this function and fills the boundary nodes appropriately. Examples:

- Dirichlet:  $A = 0$ ,  $B = 1$ . For homogeneous,  $C = 0$ .
- Neumann:  $A = 1$ ,  $B = 0$ . For homogeneous,  $C = 0$ .

The user needs to define an instance of `DomainNodeBC` only once, regardless of the number of levels of refinement. The problem domain and the mesh spacing for the level are inputs to the functions that set and apply the boundary conditions.

### 5.1 The `DomainNodeBC` class

There is one instance of `DomainNodeBC` for the whole problem domain.

- `DomainNodeBC()`  
This constructor defines the object in an unusable state until the user calls `setFaceNodeBC` for each face.
- `void setFaceNodeBC(const FaceNodeBC& a_bc)`  
Sets boundary conditions at a face.  
**Input:**

– a\_bc is boundary condition at a face.

- `void applyHomogeneousBCs(`  
    `NodeFArrayBox& a_state,`  
    `const ProblemDomain& a_domain,`  
    `Real a_dx) const`
- `void applyHomogeneousBCs(`  
    `NodeFArrayBox& a_state,`  
    `const Box& a_domain,`  
    `Real a_dx) const`
- `void applyInhomogeneousBCs(`  
    `NodeFArrayBox& a_state,`  
    `const ProblemDomain& a_domain,`  
    `Real a_dx) const`
- `void applyInhomogeneousBCs(`  
    `NodeFArrayBox& a_state,`  
    `const Box& a_domain,`  
    `Real a_dx) const`

Apply boundary conditions. The difference between `applyHomogeneousBCs` and `applyInhomogeneousBCs` is that `applyHomogeneousBCs` sets  $C = 0$  in (5.1).

**Inputs:**

- a\_domain is the cell-centered problem domain on the level.
- a\_dx is the mesh spacing on the level.

**Output:**

- a\_state contains the data. Data at nodes on the boundary of the problem domain are modified according to the boundary conditions. Data at other nodes are unchanged.

## 5.2 The FaceNodeBC interface

There is a `FaceNodeBC` object for each of the  $2 * \text{SpaceDim}$  faces of the problem domain.

- `FaceNodeBC(`  
    `int a_dir,`  
    `Side::LoHiSide a_sd)`

- FaceNodeBC(
 

```

int          a_dir,
Side::LoHiSide a_sd,
const Interval& a_comps)

```

Constructor that defines boundary conditions on a face.

**Inputs:**

- a\_dir is dimension of face, 0 up to SpaceDim-1.
- a\_sd is either Side::Lo or Side::Hi in specifying the face.
- a\_comps, if present, specifies the interval over which components of the data will have boundary values set. If absent, then the interval is (0:0).

- virtual FaceNodeBC\* new\_boxBC() const = 0

Returns a pointer to a new instance of FaceNodeBC of the same type.

- void define(
 

```

int          a_dir,
Side::LoHiSide a_sd)

```

- void define(
 

```

int          a_dir,
Side::LoHiSide a_sd,
const Interval& a_comps)

```

Define the FaceNodeBC object with the same arguments as the constructors.

- virtual void applyHomogeneousBCs(
 

```

FArrayBox&          a_state,
const ProblemDomain& a_domain,
Real                a_dx) const

```

- virtual void applyHomogeneousBCs(
 

```

FArrayBox& a_state,
const Box& a_domain,
Real        a_dx) const

```

- virtual void applyInhomogeneousBCs(
 

```

FArrayBox&          a_state,
const ProblemDomain& a_domain,
Real                a_dx) const

```

- virtual void applyInhomogeneousBCs(
 

```

FArrayBox& a_state,
const Box& a_domain,

```

Real a\_dx) const

The difference between `applyHomogeneousBCs` and `applyInhomogeneousBCs` is that `applyHomogeneousBCs` sets  $C = 0$  in (5.1).

**Inputs:**

- `a_domain` is the cell-centered problem domain on the level.
- `a_dx` is the mesh spacing on the level.

**Output:**

- `a_state` contains the data. Data on nodes of `a_state` that lie on the boundary of the cell-centered problem domain `a_domain` are modified according to the boundary conditions. Data on other nodes of `a_state` are unchanged.

- virtual void fillBCValues(  
    FArrayBox& a\_neumfac,  
    FArrayBox& a\_dircfac,  
    FArrayBox& a\_inhmval,  
    Real a\_dx,  
    const ProblemDomain& a\_domain) const = 0
- virtual void fillBCValues(  
    FArrayBox& a\_neumfac,  
    FArrayBox& a\_dircfac,  
    FArrayBox& a\_inhmval,  
    Real a\_dx,  
    const Box& a\_domain) const = 0

Sets the coefficients in (5.1). This function must be provided in the derived class.

**Inputs:**

- `a_dx` is the mesh spacing on the level.
- `a_domain` is the cell-centered problem domain on the level.

**Outputs:** Data in these node-centered `FArrayBoxes` are set on the nodes that lie on this face of the boundary of `a_domain`. When called by `applyEitherBCs`, all three of them have the same underlying node-centered box, which will be a subbox of this boundary face. In the boundary condition equation  $A \frac{\partial \varphi}{\partial n} + B\varphi = C$  (5.1):

- `a_neumfac` contains  $A$ .
- `a_dircfac` contains  $B$ .
- `a_inhmval` contains  $C$ .



# Bibliography

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- [McC02] P. McCorquodale. AMRNodeElliptic Software Package: Node-Centered AMR for Elliptic Problems. unpublished, 2002.